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Flexural strength analysis of brittle materials

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The relation between flaw density and strength of ceramic or brittle materials were derived and applied to a commonly used testing method of four-point bending for determining the strengths of ceramic or brittle materials. Previous analysis failed to include the failures outside or at the inner loading positions for four-point test data. The present approach elevates such a constrain so that the relation applies to failures occurs at any points between the outer loading positions for four-point test data.

1. Introduction

It is well known that the strength of ceramic or brittle materials is a statistical quantity, which is sensitive to intrinsic flaws. As opposed to the conventional concepts used for metallic material, statistical analysis of fracture in ceramic materials has been found to be more appropriate [1]. One of the approaches has been the popular Weibull analysis, which assumes an arbitrary function for flaw strength distribution [2]. However, real strength distribution of ceramic materials are not necessarily characterized by the Weibull distribution. In Batdorf's Approach, a crack density function related to material and stress is used [3, 4]. A Taylor series was initial proposed for the expression of the crack density. This resulted in computational difficulties. In a revised form, for analyzeing four-point bending test data, the two-parameter Weibull distribution was used to express the failure data. In elemental strength approaches [5, 6] based on non-coplanar failure criterion, prediction on biaxial strength has been found to accurate. Another generalized approach base on weakest link theory was proposed to evaluate data of uniaxial strength test, expandable tensile test, three-point bending test, and four-point bending test [7]. However, the analysis for four-point test data was based on constrains that exclude failures outside or at the inner loading positions. This paper extends this approach to include the failures outside or at the inner loading positions for four-point test data. The significance is that the role of using a standard four-point bending test for ceramic materials has been addressed [8, 9]. Specimens of sintered alumina were tested and the results were interpreted by the current analysis.

2. Deriviation

Based on the Poisson postulates [10], the probability of failure, P, of a part with volume V can be determined as follows.

$$1 - P(S) = \exp\left[-\int_V \mathrm{d}v \int_0^S g(S)\,\mathrm{d}S\right] \tag{A}$$

where $g(S)\,\mathrm{d}S$ is the number of flaws per unit volume, $\mathrm{d}v$. The purpose of a statistical analysis is to derive $g(S)$ from testing data or the probability of failure, P. The function $g(S)$ can then be used to estimate the fracture probability of components fabricated from the same material. For the popular four-point bending test, the analysis is given below. The analysis assumes that volume or internal flaws dominate. Similar approach can be shown when surface flaws dominate.

The tensile stress distribution in a specimen subjected to four-point bending, as shown in Fig. 1, can be expressed as

$$S = S_m \frac{x}{d} \tag{B}$$

and

$$S = S_m\left(\frac{x}{d}\right)\left(\frac{l_2 - y}{l_2 - l_1}\right) \tag{C}$$

for $y < l_1$ and $l_1 < y < l_2$, respectively, where S is the stress function, S_m is the maximum stress at which specimen breaks, and x, y, l_1, l_2, and d are given in Fig. 1. For volume or internal flaws, it can be shown that [11]

$$G(S_m) = -\ln[1 - P(S_m)] = \int_V \int_0^{S_m} g(S)\,\mathrm{d}S\,\mathrm{d}v \tag{D}$$

Upon substituting Eqs. (B) and (C) into Eq. (D), and rearrangement, one can obtain

$$G(S_m) = 2b\left[\int_0^{l_1}\int_0^d\int_0^{S_m} g(S)\,\mathrm{d}S\,\mathrm{d}x\,\mathrm{d}y\right] + 2b\left[\int_{l_1}^{l_2}\int_0^d\int_0^{S_m\left(\frac{x}{d}\right)\left(\frac{l_2-y}{l_2-l_1}\right)} g(S)\mathrm{d}S\,\mathrm{d}x\,\mathrm{d}y\right] \tag{E}$$

where b is the width of the specimen. The above equation can be further simplified to

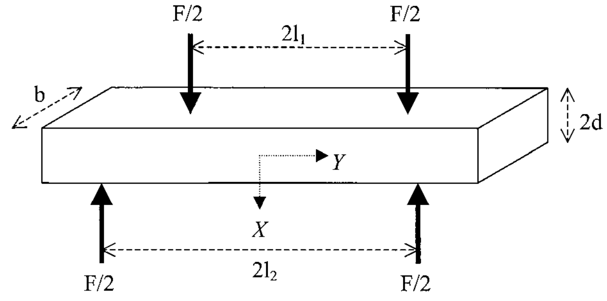

Figure 1 A four-point bending test for ceramic or brittle materials.

$$G(S_m) = \frac{A}{S_m}\int_0^{S_m} Sg(S)\,\mathrm{d}S - \frac{B}{S_m}\int_0^{S_m} S\ln\left(\frac{S}{S_m}\right)g(S)\,\mathrm{d}S \tag{F}$$

where $A = 2bdl_1$ and $B = 2bd(l_2 - l_1)$. Taking the first and second derivatives of $G(S_m)$, one obtains

$$G'(S_m) = Ag(S_m) - \frac{(A-B)}{S_m^2}\int_0^{S_m} Sg(S)\,\mathrm{d}S + \frac{B}{S_m^2}\int_0^{S_m} S\ln\left(\frac{S}{S_m}\right)g(S)\,\mathrm{d}S \tag{G}$$

and

$$G''(S_m) = -\frac{(A-B)}{S_m}g(S_m) + A'g(S_m) + \frac{(2A-3B)}{S_m^3}\int_0^{S_m} Sg(S)\,\mathrm{d}S - \frac{2B}{S_m^3}\int_0^{S_m} S\ln\left(\frac{S}{S_m}\right)g(S)\,\mathrm{d}S \tag{H}$$

where $g'(S_m)$ is the first derivative of $g(S_m)$. From Eqs. (F) and (G), one can have

$$\int_0^{S_m} Sg(S)\,\mathrm{d}S = \frac{S_m}{B}[G(S_m) + S_m G'(S_m) - AS_m g(S_m)] \tag{I}$$

and

$$\int_0^{S_m} S\ln\left(\frac{S}{S_m}\right)g(S)\,\mathrm{d}S = \frac{S_m}{B}\left[\left(\frac{A-B}{B}\right)G(S_m) + \frac{A}{B}S_m G'(S_m) - \frac{A^2}{B}S_m g(S_m)\right] \tag{J}$$

Substitution of Eqs. (I) and (J) into Eq. (H) gives the following first order differential equation of $g(S_m)$.

$$g(S_m) + \frac{l_1 S_m}{l_1 + l_2}g'(S_m) = \frac{1}{2bd(l_1+l_2)} \times \left[\frac{G(S_m)}{S_m} + 3G'(S_m) + S_m G''(S_m)\right] \tag{K}$$

The general solution for the above first-order differential equation is

$$g(S_m) = \exp\left[\frac{-l_1 S_m^2}{2(l_1+l_2)}\right]\left(\int_0^{S_m} \exp\left[\frac{l_1 S^2}{2(l_1+l_2)}\right] \times \left[\frac{1}{2bd(l_1+l_2)}\left(\frac{G(S)}{S} + 3G'(S) + SG''(S)\right)\right]\mathrm{d}S + C\right) \tag{L}$$

where C is a constant. In the above equation, $g(S_m)$ can be determined by data fitting of P or G, which can be obtained through testing a sufficient amount of specimens.

3. Data analysis

Three high-purity sintered alumina, designated as BC(N), BR(N), and BA, were prepared for four-point bending test. Characteristics of these alumina are given in Table I. The four-point bending test was conducted according to Military Standard 1942(MR) [8]. The specimen dimensions were 50.0 mm by 4.0 mm (b in Fig. 1) by 3.0 mm (2d in Fig. 1). The inner span, $2l_1 = 20.0$ mm and the outer span, $2l_2 = 40.0$ mm. The surface of each specimen was polished with a 600-grit diamond wheel. The four long edges of each specimen were uniformly chamfered ar 45°, a distance of 0.15 mm. The test was performed at a cross-head speed of 0.05 cm/s. For each material, at least 50 specimens were tested. Examination on the fractured specimens indicates that failures occurred mostly inside the inner loading positions while some were at and outside the inner loading positions. Data were taken only from the specimens that fractured inside the inner loading positions.

Data from the bending test were first ordered. The cumulative probability of failure, $P(S_m)$, was then plotted as a function of failure or peak stress. This is shown in Fig. 2 for all the materials. Not all the data points are shown as many of them merge to form a single point on the present scale. The relation between P and G given in Eq. (D) was then determined and is shown in Fig. 3. As shown in Fig. 3, no significant localized peak in $G(S_m)$ was found. This allows polynomial fits to be obtained for function $G(S_m)$ in a way that local oscillations are smoothed while the general shape is retained. Therefore, the derivatives of $G(S_m)$, which determine $g(S_m)$, can be deduced from the polynomials. Such polynomial fits for all three test sets are shown in Fig. 3 in terms of the function, $G(S_m)$, The flaw densities for all three materials can therefore be obtained

TABLE I Density, porosity, and average grain size of sintered alumina

Characteristics \ Material	BA	BR(N)	BC(N)
Density (g/cc)	3.89	3.94	3.89
Porosity (%)	2.51	1.01	2.26
Grain size (μm)	1.43	1.66	2.53

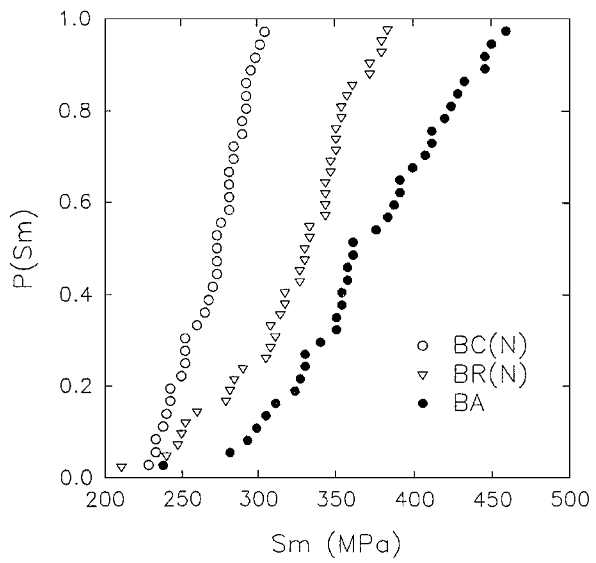

Figure 2 Failure probability as a function of strength for all three materials studied.

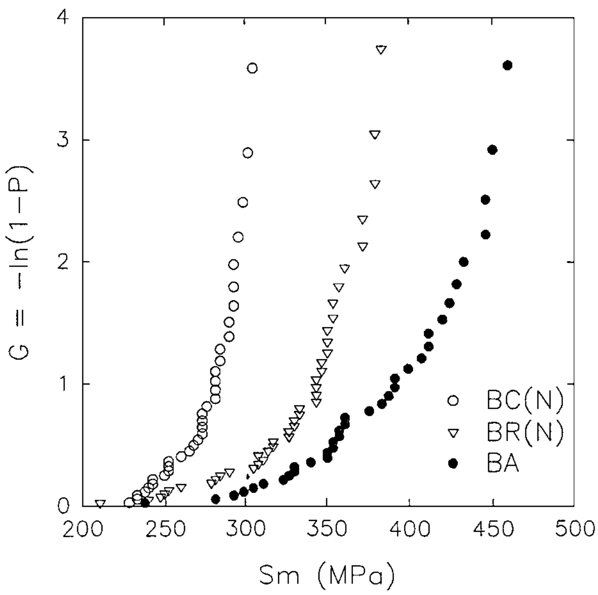

Figure 3 Polynomial fits to the data shown in Fig. 2 where the function $G(S_m)$ is used.

using the polynomials in Fig. 3 and Eq. (M). The result is shown in Fig. 4. Comparing all three materials, Material BC(N) has the highest flaw densities at relatively low values of strength. On the other hand, Material BA exhibits the lowest flaw densities at relatively high values of strength. This result, together with the data given in Table I, suggest that in these materials a grain can be regarded as a flaw in many brittle polycrystalline materials [12].

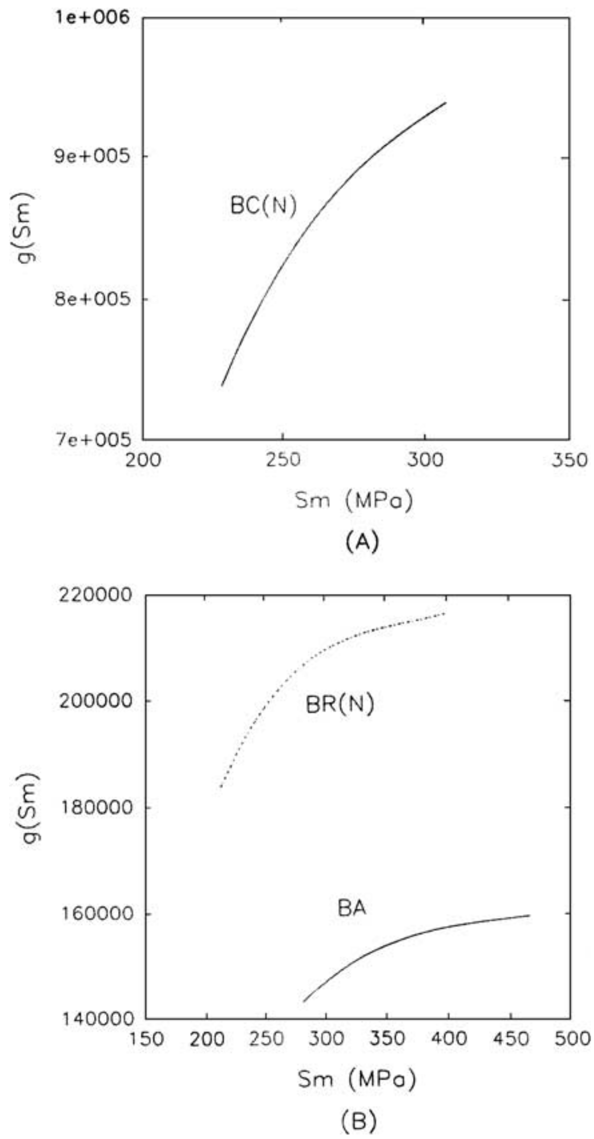

Figure 4 Flaw density obtained from Fig. 3 and Eq. (M). $g(S_m)$ in $(\mathrm{Mnm})^{-1}$.

4. Conclusion

Previous statistical approaches for analyzing the strength of brittle materials fail to include the failures outside or at the inner loading positions for four-point test data. The present approach elevates such a constrain, and a formulation was derived and applied to failures occurs at any points between the outer loading positions for four-point test data.

References

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